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1 Length

To base vector multiplication on length, and examine all the consequences of having done so, it is first necessary to The geometrical definition of length for vectors generalizes Pythagoras theorem to higher dimensions.

In two dimensions this theorem can be proved with the aid of the following diagram

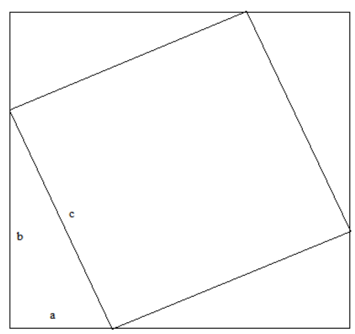

Figure 1: Geometrical Proof of Pythagoras Theorm for Right Triangle

The area of the interior and exterior squares is c^2, and $(a+b)^2$ respectively. The interior area can also be calculated by subtracting the area of the triangles from the exterior area:

$$(a+b)^2 - 4(ab/2) = a^2 + b^2 + 2ab - 2ab$$

Thus proving Phythagoras theorem for the length of the diagonal in a right angle triangle

$$c^2 = a^2 + b^2$$

The length of a vector in three dimensions can be found by repeated application of Pythagoras theorem, as in the following figure

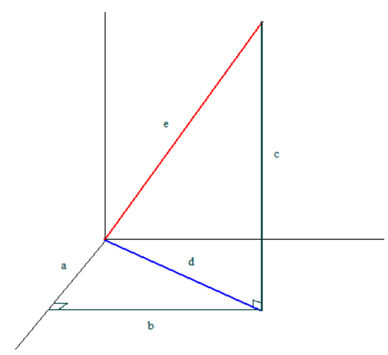

Figure 2: Length of vector in three dimensions

The vector $\mathbf{e} = \mathbf{a} + \mathbf{b} + \mathbf{c}$, where each of the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are mutually perpendicular can be found by first calculating

$$d^2 = a^2 + b^2$$

Then

$$e^2 = d^2 + c^2 = a^2 + b^2 + c^2$$

This process can be repeated for any number of higher dimensions. Having calculated the length of a $N - 1$ dimensional vector

$$L(\mathbf{v})^2 = \sum_{i=1}^{N-1} l_i{}^2$$

Once an additional component of length l_N is added to that vector in a direction mutually perpendicular to all previous components the new length of this vector becomes

$$\sum_{i=1}^{N-1} l_i{}^2 + l_N^2 = \sum_{i=0}^{N} l_i{}^2$$

This is what we mean by the geometrical length of a vector.