Some notes on DeBroglie paper.

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1 Motivation.

The translation of the DeBroglie thesis [Kracklauer.()] appears to have a quite readable introduction to many relativity and quantum phenomina. Here I collect additional notes on things that were not clear to me.

2 Chapter 2.

2.1 Equation 2.2.10

Let

$$\mathcal{L} = k = g_{ij}\dot{q}^i\dot{q}^j$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}^k} = 2g_{ik}\dot{q}^i$$

$$\frac{\partial \mathcal{L}}{\partial \dot{q}^k}\dot{q}^k = 2g_{ik}\dot{q}^i\dot{q}^k = 2K$$

2.2 Equation 2.3.3

A worldline velocity with respect to some parameterization is

$$\left(\frac{ds}{d\lambda}\right)^2 = \left(\frac{dx}{d\lambda}\cdot\frac{dx}{d\lambda}\right)^2 = \left(\frac{dx^4}{d\lambda}\right)^2 - \sum_i \left(\frac{dx^i}{d\lambda}\right)^2$$

For $\lambda = s$, we can therefore calculate u^4:

$$
\begin{aligned}
\left(\frac{ds}{ds}\right)^2 = 1 = \left(\frac{dx^4}{ds}\right)^2 - \sum_i \left(\frac{dx^i}{ds}\right)^2 \\
&= \left(\frac{dx^4}{ds}\right)^2 - \sum_i \left(\frac{dx^i}{dx^4}\frac{dx^4}{ds}\right)^2 \\
&= \left(\frac{dx^4}{ds}\right)^2 \left(1 - \sum_i \left(\frac{dx^i}{dx^4}\right)^2\right)
\end{aligned}
$$

Or

$$
\left(\frac{dx^4}{ds}\right)^2 = \frac{1}{1 - \mathbf{v}^2/c^2} \tag{1}
$$

$$
\tag{2}
$$

There is a freedom to pick either plus or minus here. Returning to that later, first calcuate the remainder of this table of derivatives. Picking x^1 as representitive

$$
\begin{aligned}
1 &= \frac{1}{1 - \mathbf{v}^2/c^2} - \left(\frac{dx^1}{ds}\right)^2 - \left(\frac{dy}{dx^4}\frac{dx^4}{ds}\right)^2 - \left(\frac{dz}{dx^4}\frac{dx^4}{ds}\right)^2 \\
\left(\frac{dx^1}{ds}\right)^2 &= \frac{\mathbf{v}^2/c^2}{1 - \mathbf{v}^2/c^2} - \frac{1}{1 - \mathbf{v}^2/c^2}\left(\left(\frac{dy}{dx^4}\right)^2 + \left(\frac{dz}{dx^4}\right)^2\right) \\
&= \frac{1}{c^2(1 - \mathbf{v}^2/c^2)}\left(\mathbf{v}^2 - \left(\frac{dy}{dt}\right)^2 - \left(\frac{dz}{dt}\right)^2\right) \\
&= \frac{v_x^2/c^2}{1 - \mathbf{v}^2/c^2}
\end{aligned}
$$

Now, express the coordinate vector for the worldline differential in its entirety:

$$
\begin{aligned}
\frac{dx}{ds} = \frac{d}{ds}(x^1, x^2, x^3, x^4) &= \frac{dx^4}{ds}\left(\frac{dx^1}{dx^4}, \frac{dx^2}{dx^4}, \frac{dx^3}{dx^4}, 1\right) \\
&= \frac{dx^4}{ds}(v_x/c, v_y/c, v_z/c, 1)
\end{aligned}
$$

This shows that the flexibilty to choose a sign for the square roots to obtain dx^μ/ds must all match the sign for the dx^4/ds term. Considering a particle at rest in the implied frame associated with these coordinates, one has, by 1

$$
\begin{aligned}
\frac{dx}{ds} &= \pm\left(0, 0, 0, \frac{1}{\sqrt{1-\mathbf{v}^2/c^2}}\right) \\
&= \pm\,(0, 0, 0, 1)
\end{aligned}
$$

If we take positive ds to measure increase of time in the rest frame, then there is some sense to picking the positive root. One wouldn't have to, since there is also a corresponding freedom to bury a sign adjustment in the dx_μ/ds derivatives.

References

[Kracklauer.()] A. F. Kracklauer. Louis de broglie (thesis): On the theory of quanta. ″http://www.nonloco-physics.000freehosting.com/ldb_the.pdf″.