

Rotor interpolation calculation.

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The aim is to compute the interpolating rotor r that takes an object from one position to another in n steps. Here the initial and final positions are given by two rotors R_1 , and R_2 like so

$$\begin{aligned}X_1 &= R_1 X R_1^\dagger \\X_2 &= R_2 X R_2^\dagger = r^n R_1 X R_1^\dagger r^{n\dagger}\end{aligned}$$

So, writing

$$a = r^n = R_2 \frac{1}{R_1} = \frac{R_2 R_1^\dagger}{R_1 R_1^\dagger} = \cos \theta + I \sin \theta$$

So,

$$\begin{aligned}\frac{\langle a \rangle_2}{\langle a \rangle} &= \frac{\langle a \rangle_2}{|\langle a \rangle_2|} \frac{|\langle a \rangle_2|}{\langle a \rangle} \\ &= I \tan \theta\end{aligned}$$

Therefore the interpolating rotor is:

$$\begin{aligned}I &= \frac{\langle a \rangle_2}{|\langle a \rangle_2|} \\ \theta &= \text{atan2}(|\langle a \rangle_2|, \langle a \rangle) \\ r &= \cos(\theta/n) + I \sin(\theta/n)\end{aligned}$$