Bohm Chapter 9 problems.

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Contents

1 Bohm Chapter 9 problems.

Problems and additional details from reading of [Bohm(1989)], chapter 9.

1.1 P1. Momentum wave function normalization.

Given a normalized wave function

$$\int_{-\infty}^{\infty} \psi^*(x)\psi(x)dx = 1$$

Show that the wave function $\phi(k)$ is also normalized, and find the normalization factor for $\Phi(p)$.

$$\int_{-\infty}^{\infty} \phi^*(k)\phi(k)dk = \int_{-\infty}^{\infty} \phi^*(k)\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi(x)e^{-ikx}dx\right)dk$$

$$= \int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi^*(k)e^{-ikx}dk\right)\psi(x)dx$$

$$= \int_{-\infty}^{\infty}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(k)e^{ikx}dk\right)^*\psi(x)dx$$

$$= \int_{-\infty}^{\infty}\psi^*(x)\psi(x)dx$$

$$= 1 \qquad \square$$

Bohm defines $\Phi(p) \propto \phi(k)$ with the normalization constant determined by $\int |\Phi(p)|dp = 1$. Suppose we let $\Phi(p) = \alpha\phi(k)$, then we have

$$1 = \int \Phi^*(p)\Phi(p)dp$$

$$= \int \alpha^2\phi^*(k)\phi(k)\hbar dk$$

So we want $\alpha^2\hbar = 1$, and therefore $\Phi(p) = \frac{1}{\sqrt{\hbar}}\phi(k)$.

In [McMahon(2005)], with followup in [Joot()] we've seen that an alternate Fourier transform pair can be used in terms of momentum variables. That is

$$\Phi(p) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\psi(x)e^{-ipx/\hbar}dx$$

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\Phi(p)e^{ipx/\hbar}dp$$

Observe that this is consistent with Bohm's notation, since one can read off $\Phi(p)$ in terms of $\phi(k)$. by inspection

$$\Phi(p) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}\psi(x)e^{-ipx/\hbar}dx = \frac{1}{\sqrt{\hbar}}\phi(k)$$

1.2 P2. Expectation of polynomial momentum function.

Given a function of momentum

$$f(p) = \sum C_n p^n$$

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Express the average, or expectation value of $f(p)$. It is sufficient to consider one of the monomial terms, say p^n. A translation to position basis via Fourier transformation produces the desired result

$$
\begin{aligned}
\langle p^n \rangle &= \int \Phi^*(p) p^n \Phi(p) dp \\
&= \frac{1}{2\pi\hbar} \iiint \left(\psi^*(x') e^{ipx'/\hbar} dx' \right) (\hbar k)^n \left(\psi(x) e^{-ipx/\hbar} dx \right) (\hbar dk) \\
&= \frac{\hbar^n}{2\pi} \iiint \psi^*(x') e^{ikx'} dx' k^n e^{-ikx} \psi(x) dx dk
\end{aligned}
$$

The k^n can be reduced to differential form as Bohm did for the $\langle p \rangle$ case

$$
\begin{aligned}
k^n e^{-ikx} &= k^{n-1} k e^{-ikx} \\
&= k^{n-1} i \frac{\partial}{\partial x} e^{-ikx} \\
&= k^{n-m} i^m \frac{\partial^m}{\partial x^m} e^{-ikx} \\
&= i^n \frac{\partial^n}{\partial x^n} e^{-ikx}
\end{aligned}
$$

This leaves something that's in shape for integration by parts

$$
\begin{aligned}
\langle p^n \rangle &= \frac{(i\hbar)^n}{2\pi} \iiint \psi^*(x') e^{ikx'} dx' \left(\frac{\partial^n}{\partial x^n} e^{-ikx} \right) \psi(x) dx dk \\
&= \frac{(-i\hbar)^n}{2\pi} \iiint \psi^*(x') e^{ikx'} dx' \frac{\partial^n \psi(x)}{\partial x^n} e^{-ikx} dx dk \\
&= \frac{(-i\hbar)^n}{2\pi} \iiint \psi^*(x') e^{ik(x'-x)} \frac{\partial^n \psi(x)}{\partial x^n} dx' dx dk \\
&= (-i\hbar)^n \iint \psi^*(x') \frac{\partial^n \psi(x)}{\partial x^n} dx' dx \frac{1}{2\pi} \int e^{ik(x'-x)} dk
\end{aligned}
$$

This last integral is really a distribution, and can be identified with the delta function $\delta(x' - x)$ operating on, in this case, the preceding integral.

So we have

$$
\begin{aligned}
\langle p^n \rangle &= (-i\hbar)^n \iint \psi^*(x') \frac{\partial^n \psi(x)}{\partial x^n} dx' dx \delta(x' - x) \\
&= (-i\hbar)^n \int \psi^*(x) \frac{\partial^n \psi(x)}{\partial x^n} dx
\end{aligned}
$$

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We can put this into explicit operator form, nicely motivating the identification of $-i\hbar\partial/\partial x$ with the momentum by virtue of the definition of the average or expectation value.

$$\langle p^n \rangle = \int \psi^*(x) \left(-i\hbar \frac{\partial}{\partial x} \right)^n \psi(x) dx$$

1.3 P3. Expectation of position in momentum space.

$$\begin{aligned}
\langle x \rangle &= \int \psi^*(x) x \psi(x) dx \\
&= \frac{1}{2\pi\hbar} \iiint \Phi^*(p) e^{-ipx/\hbar} dp\, x \Phi(p') e^{ip'x/\hbar} dp' dx \\
&= \frac{1}{2\pi\hbar} \iiint \Phi^*(p) e^{-ipx/\hbar} dp \left(-i \frac{\partial}{\partial p'} e^{ip'x/\hbar} \right) \Phi(p') dp' dx \\
&= \frac{1}{2\pi\hbar} \iiint \Phi^*(p) e^{-ipx/\hbar} dp \left(i \frac{\partial \Phi(p')}{\partial p'} \right) e^{ip'x/\hbar} dp' dx \\
&= \iint \Phi^*(p) \left(i \frac{\partial}{\partial p'} \right) \Phi(p') dp dp' \frac{1}{2\pi\hbar} \int e^{i(p'-p)x/\hbar} dx \\
&= \iint \Phi^*(p) \left(i \frac{\partial}{\partial p'} \right) \Phi(p') dp dp' \delta(p' - p)
\end{aligned}$$

This is

$$\langle x \rangle = \int \Phi^*(p) \left(i \frac{\partial}{\partial p} \right) \Phi(p) dp$$

We see that expressing momentum in position space and position in momentum space both result in differential operator forms in calculations of expected values

$$p \sim -i\hbar \frac{\partial}{\partial x} \tag{1}$$

$$x \sim i\hbar \frac{\partial}{\partial p} \tag{2}$$

Observe the Hamiltonian and Poisson equation structure in these two sets of operators.

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1.4 P4. Expectation of polynomial position function.

This problem follows just as P2, and I'm not going to bother typing it up for myself. For validity, we require $x^n \phi(x) \to 0$ as $x \to \pm\infty$, or equivalently that $\frac{\partial^n \Phi}{\partial p^n} \to 0$.

1.5 P5. Some commutator calculations.

1.5.1 P5. Position momentum moment commutators.

Evaluate

$$f(x, p) = x^n p^m - p^m x^n$$

Up to this point we've only seen operators in expectation values. Let's look the simplest case with $n = m = 1$ in that context

$$\langle f \rangle = \frac{\hbar}{i} \int \psi^*(x) \left(x \frac{\partial}{\partial x} - \frac{\partial}{\partial x} x \right) \psi(x) dx$$

$$= \frac{\hbar}{i} \int \psi^*(x) \left(x \frac{\partial \psi(x)}{\partial x} - \psi(x) - x \frac{\partial \psi(x)}{\partial x} \right) dx$$

$$= -\frac{\hbar}{i} \int \psi^*(x) \psi(x) dx$$

$$= i\hbar$$

So in the same way that the operator correspondence between momentum and the derivative as summarized in 1, one can associate the commutator operator with its action in the expectation value and say

$$xp - px \sim i\hbar \tag{3}$$

The higher order commutator expansions could also be evaluated this way, but exploiting the operator nature directly makes this easier. For the first order moment commutator above one can write

$$f(x, p)\psi(x) = (xp - px)\psi(x)$$

$$= -i\hbar \left(x \frac{\partial}{\partial x} - \frac{\partial}{\partial x} x \right) \psi(x)$$

$$= -i\hbar \left(x \frac{\partial \psi}{\partial x}(x) - \frac{\partial x \psi(x)}{\partial x} \right)$$

$$= -i\hbar \left(x \frac{\partial \psi}{\partial x}(x) - \frac{\partial \psi(x)}{\partial x} - \psi(x) \right)$$

$$= i\hbar \psi(x)$$

So again we see that as a right acting operator the net effect on any wave function is the following action

$$(xp - px)\psi = i\hbar\psi$$

If one starts from this point and then calculates the expectation value the result will still be $i\hbar$, but working with the probability integrals from the get go is just additional complication.

Building on this result we can then calculate the higher order moment differences of the problem by using the commutator to change the order of operations

$$px \sim -i\hbar + xp$$

Let's use this for a couple simple examples to start

$$
\begin{aligned}
x^2 p - px^2 &= x^2 p - (-i\hbar + xp)x \\
&= x^2 p + i\hbar x - x(-i\hbar + xp) \\
&= x^2 p + 2i\hbar x - x^2 p \\
&= 2i\hbar x
\end{aligned}
$$

$$
\begin{aligned}
xp^2 - p^2 x &= xp^2 - p(-i\hbar + xp) \\
&= xp^2 + i\hbar p - pxp \\
&= xp^2 + i\hbar p + (+i\hbar - xp)p \\
&= 2i\hbar p
\end{aligned}
$$

Calculation of third powers shows a pattern, and one can guess at an induction hypothesis

$$
\begin{aligned}
xp^n &= p^n x + ni\hbar p^{n-1} \\
-px^n &= -x^n p + ni\hbar x^{n-1}
\end{aligned}
$$

The $n = 1$ cases follow from $xp - px = i\hbar$, leaving only the induction on n. For the momentum powers we have

$$xp^n p = p^n xp + ni\hbar p^n$$
$$= p^n(px + i\hbar) + ni\hbar p^n$$
$$= p^{n+1}x + (n+1)i\hbar p^n \qquad \square$$

For the position powers we have

$$-px^n x = -x^n px + ni\hbar x^n$$
$$= x^n(-xp + i\hbar) + ni\hbar x^n$$
$$= -x^{n+1}p + (n+1)i\hbar x^n \qquad \square$$

This completes the proof for a first order version of the problem

$$xp^n - p^n x = ni\hbar p^{n-1}$$
$$x^n p - px^n = ni\hbar x^{n-1}$$

Observe that working with the operator form changes the calculation of derivatives problem in the original commutator evaluation to nothing more than an algebraic exercise.

The general case still remains. Building up to that let's do a couple examples

$$x^n p^2 = (x^n p)p$$
$$= (px^n + ni\hbar x^{n-1})p$$
$$= p(x^n p) + ni\hbar(x^{n-1}p)$$
$$= p^2 x^n + 2ni\hbar px^{n-1} + n(n-1)(i\hbar)^2 x^{n-2}$$

$$x^n p^3 = (x^n p^2)p$$
$$= (p^2 x^n + 2ni\hbar px^{n-1} + n(n-1)(i\hbar)^2 x^{n-2})p$$
$$= p^2(px^n + ni\hbar x^{n-1}) + 2ni\hbar p(px^{n-1} + (n-1)i\hbar x^{n-2}) + n(n-1)(i\hbar)^2(px^{n-2} + (n-2)i\hbar x^{n-3})$$
$$= p^3 x^n + 3n(i\hbar)p^2 x^{n-1} + 3n(n-1)(i\hbar)^2 px^{n-2} + n(n-1)(n-2)(i\hbar)^3 x^{n-3}$$

We see what looks like binomial coefficients, so a reasonable inductive hypothesis, for $m \leq n$

$$x^n p^m = \sum_{j=0}^{m} \binom{m}{j} (i\hbar)^j p^{m-j} x^{n-j} (n)(n-1)\cdots(n-j+1) \qquad (4)$$

And in particular, for $m \le n$

$$x^n p^m - p^m x^n = \sum_{j=1}^{m} \binom{m}{j} (i\hbar)^j p^{m-j} x^{n-j} (n)(n-1)\cdots(n-j+1) \qquad (5)$$

For $m \ge n$, let's start with

$$p^m x = x p^m - m i\hbar p^{m-1}$$

First do the x^2

$$\begin{aligned}
p^m x^2 &= x p^m x - m(i\hbar) p^{m-1} x \\
&= x(p^m x) - m(i\hbar)(p^{m-1} x) \\
&= x^2 p^m - 2m(i\hbar) x p^{m-1} + m(m-1)(i\hbar)^2 p^{m-2}
\end{aligned}$$

And for the cube x^3

$$\begin{aligned}
p^m x^3 &= (p^m x^2) x \\
&= (x^2 p^m - 2m(i\hbar) x p^{m-1} + m(m-1)(i\hbar)^2 p^{m-2}) x \\
&= x^2 (p^m x) - 2m(i\hbar) x (p^{m-1} x) + m(m-1)(i\hbar)^2 (p^{m-2} x) \\
&= x^2 (x p^m - m(i\hbar) p^{m-1}) \\
&\quad - 2m(i\hbar) x (x p^{m-1} - (m-1)(i\hbar) p^{m-2}) \\
&\quad + m(m-1)(i\hbar)^2 (x p^{m-2} - (m-2)(i\hbar) p^{m-3}) \\
&= x^3 p^m - 3m(i\hbar) x^2 p^{m-1} + 3m(m-1)(i\hbar)^2 x p^{m-2} - m(m-1)(i\hbar)^2 (m-2)(i\hbar) p^{m-3}
\end{aligned}$$

It appears that in this case with $m \ge n$, like 4, we want as the induction statement

$$p^m x^n = \sum_{j=0}^{n} \binom{n}{j} (-i\hbar)^j x^{n-j} p^{m-j} (m)(m-1)\cdots(m-j+1) \qquad (6)$$

And for the commutator moment the expected result, pending induction on the above, is

$$x^n p^m - p^m x^n = -\sum_{j=1}^{n} \binom{n}{j} (-i\hbar)^j x^{n-j} p^{m-j} (m)(m-1)\cdots(m-j+1) \qquad (7)$$

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Summarizing, this is

$$x^n p^m - p^m x^n =$$
$$\begin{cases} \sum_{j=1}^{m} \binom{m}{j}(i\hbar)^j p^{m-j} x^{n-j}(n)(n-1)\cdots(n-j+1) & \text{if } m \leq n \\ -\sum_{j=1}^{n} \binom{n}{j}(-i\hbar)^j x^{n-j} p^{m-j}(m)(m-1)\cdots(m-j+1) & \text{if } m \geq n \end{cases}$$

1.5.2 P5.b

$$e^{ikx} p - p e^{ikx}$$

Reversing the second term via power series expansion we have

$$\begin{aligned} p e^{ikx} &= p \sum_{n=0}^{\infty} \frac{(ikx)^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{(ik)^n}{n!}(x^n p - ni\hbar x^{n-1}) \\ &= e^{ikx} p - \sum_{n=1}^{\infty} \frac{(ik)^n}{n!}(ni\hbar x^{n-1}) \\ &= e^{ikx} p - (ik)(i\hbar) \sum_{n=1}^{\infty} \frac{(ikx)^{n-1}}{(n-1)!} \\ &= e^{ikx} p + (k\hbar)e^{ikx} \end{aligned}$$

So we have

$$e^{ikx} p - p e^{ikx} = -(k\hbar)e^{ikx}$$

1.6 P6. Hermitian operators. Powers of momentum operators.

Show that p^n is Hermitian

$$\begin{aligned} \langle p^n \rangle^* &= \left(\int \psi^* (-i\hbar)^n \frac{d^n}{dx^n} \psi \right)^* \\ &= \int \psi (i\hbar)^n \frac{d^n}{dx^n} \psi^* \\ &= \int \left((-1)^n \frac{d^n}{dx^n} \psi \right)(i\hbar)^n \psi^* \\ &= \int (p^n \psi) \psi^* \\ &= \langle p^n \rangle \end{aligned}$$

Thus, by the definition of equation (13) in the text, this operator is Hermitian.

Next is to show that

$$f(p) = \sum_k A_k p^k$$

is Hermitian, provided A_k are all real. This part is clear by inspection.

1.7 P7. Hermitian operators. Powers of position operators.

Want to show that the following is Hermitian

$$f(x) = \sum A_k x^k$$

If the conjugate of the expectation equals itself we required only $A_k = A_k^*$, so A_k must be strictly real, and we are done.

1.8 P8. Non Hermitian momentum power operators if derivative doesn't vanish.

Show that if $\partial^n \psi / \partial x^n$ doesn't vanish then $(-i\hbar)^{n+1} \partial^{n+1} / \partial x^{n+1}$ is not Hermitian.

We want to evaluate the following and compare it to its conjugate

$$\langle p^{n+1} \rangle = (-i\hbar)^{n+1} \int_{-\infty}^{\infty} \psi^* \frac{\partial^{n+1} \psi}{\partial x^{n+1}} dx$$

$$= (-i\hbar)^{n+1} \left. \psi^* \frac{\partial^n \psi}{\partial x^n} \right|_{-\infty}^{\infty} + (-1)^1 (-i\hbar)^{n+1} \int_{-\infty}^{\infty} \frac{\partial^1 \psi^*}{\partial x^1} \frac{\partial^n \psi}{\partial x^n} dx$$

$$= (-i\hbar)^{n+1} \left. \psi^* \frac{\partial^n \psi}{\partial x^n} \right|_{-\infty}^{\infty} + (-1)^1 (-i\hbar)^{n+1} \left. \frac{\partial^1 \psi^*}{\partial x^1} \frac{\partial^{n-1} \psi}{\partial x^{n-1}} \right|_{-\infty}^{\infty} + (-1)^2 (-i\hbar)^{n+1} \int_{-\infty}^{\infty} \frac{\partial^2 \psi^*}{\partial x^2} \frac{\partial^{n-1} \psi}{\partial x^{n-1}} dx$$

$$= (-i\hbar)^{n+1} \sum_{k=0}^{1} (-1)^k \left. \frac{\partial^k \psi^*}{\partial x^k} \frac{\partial^{n-k} \psi}{\partial x^{n-k}} \right|_{-\infty}^{\infty} + (-1)^2 (-i\hbar)^{n+1} \int_{-\infty}^{\infty} \frac{\partial^2 \psi^*}{\partial x^2} \frac{\partial^{n-1} \psi}{\partial x^{n-1}} dx$$

$$= (-i\hbar)^{n+1} \sum_{k=0}^{m} (-1)^k \left. \frac{\partial^k \psi^*}{\partial x^k} \frac{\partial^{n-k} \psi}{\partial x^{n-k}} \right|_{-\infty}^{\infty} + (-1)^{m+1} (-i\hbar)^{n+1} \int_{-\infty}^{\infty} \frac{\partial^{m+1} \psi^*}{\partial x^{m+1}} \frac{\partial^{n-m)} \psi}{\partial x^{n-m)}} dx$$

$$= (-i\hbar)^{n+1} \sum_{k=0}^{n} (-1)^k \left. \frac{\partial^k \psi^*}{\partial x^k} \frac{\partial^{n-k} \psi}{\partial x^{n-k}} \right|_{-\infty}^{\infty} + (i\hbar)^{n+1} \int_{-\infty}^{\infty} \frac{\partial^{n+1} \psi^*}{\partial x^{n+1}} \psi \, dx$$

$$= (-i\hbar)^{n+1} \sum_{k=0}^{n} (-1)^k \left. \frac{\partial^k \psi^*}{\partial x^k} \frac{\partial^{n-k} \psi}{\partial x^{n-k}} \right|_{-\infty}^{\infty} + \langle p^{n+1} \rangle^*$$

So we have

$$\langle p^{n+1}\rangle - \langle p^{n+1}\rangle^* = (-i\hbar)^{n+1} \sum_{k=0}^{n} (-1)^k \left. \frac{\partial^k \psi^*}{\partial x^k} \frac{\partial^{n-k} \psi}{\partial x^{n-k}} \right|_{-\infty}^{\infty}$$

If p^{n+1} is Hermitian, then this difference should be zero, but if the indicated partial doesn't vanish this remainder bit can be non-zero.

1.9   P9. m, n'th moment is Hermitian.

Consider the operator

$$\sum A_{nm} \left(\frac{p^n x^m + x^m p^n}{2} \right)$$

Show that this is Hermitian if all A_{nm} are real.

Consider first one specific term with A_{nm}, calculate the conjugate of the expectation value, and integrate by parts

$$
\begin{aligned}
\left(\int \psi^* \frac{1}{2}(p^n x^m + x^m p^n)\psi \right)^* &= \frac{1}{2}(-1)^n \int \psi(p^n x^m + x^m p^n)\psi^* \\
&= \frac{1}{2}(i\hbar)^n \int \psi \left(\frac{d^n(x^m \psi^*)}{dx^n} + x^m \frac{d^n \psi^*}{dx^n} \right) \\
&= \frac{1}{2}(-i\hbar)^n \int x^m \psi^* \left(\frac{d^n \psi}{dx^n} + \psi^* \frac{d^n(x^m \psi)}{dx^n} \right) \\
&= \frac{1}{2} \int \psi^* (x^m p^n + p^n x^m)\psi
\end{aligned}
$$

This shows that $(p^n x^m + x^m p^n)/2$ is Hermitian, and the conjugation requires A_{nm} to be real for the product of the two to be Hermitian.

1.10   P10. Hermitizing Classical operator $(px)^2$.

Show that

$$\frac{1}{2}\left(x^2 p^2 + p^2 x^2 \right)$$

and

$$\frac{1}{4}(xp + px)^2$$

lead to results that differ by a factor of $\hbar^2$.

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To do so consider the difference of the expectation of this operator, first calculating this difference. We will want to use the commutator relation, in a few equivalent forms

$$xp - px = i\hbar$$
$$xp = px + i\hbar$$
$$px = xp - i\hbar$$
$$xp + px = 2px + i\hbar$$
$$= 2xp - i\hbar$$

This gives us

$$\frac{1}{4}(xp + px)^2 = \frac{1}{4}(2px + i\hbar)(2xp - i\hbar)$$
$$= px^2p + i\hbar\frac{1}{2}(xp - px) + \frac{1}{4}\hbar^2$$
$$= px^2p - \hbar^2\frac{1}{2} + \frac{1}{4}\hbar^2$$
$$= px^2p - \hbar^2\frac{1}{4}$$

For the other operator, reduction to a form that also contains px^2p, we have

$$\frac{1}{2}\left(x^2p^2 + p^2x^2\right) = \frac{1}{2}(x(xp)p + p(px)x)$$
$$= \frac{1}{2}(x(px + i\hbar)p + p(xp - i\hbar)x)$$
$$= \frac{1}{2}((xp)xp + px(px) + i\hbar(xp - px))$$
$$= \frac{1}{2}\left((px + i\hbar)xp + px(xp - i\hbar) + (i\hbar)^2\right)$$
$$= \frac{1}{2}\left(2px^2p + i\hbar(xp - px) + -\hbar^2\right)$$
$$= px^2p + -\hbar^2$$

So now, if we take the difference

$$\frac{1}{2}\left(x^2p^2 + p^2x^2\right) - \frac{1}{4}(xp + px)^2 = (px^2p + -\hbar^2) - (px^2p - \hbar^2\frac{1}{4})$$
$$= -\frac{3}{4}\hbar^2$$

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The difference of the expectation values of these operators is thus of the order $\hbar^2$ as was to be calculated.

1.11 P11. An explicit calculation of a Hermitian operator.

Show by integration by parts that $(xp)^\dagger = px$.

The defining relation for the Hermitian conjugation operation is equation 16 in the text.

$$\int \psi^*(O^\dagger \psi)dx = \int \psi(O^* \psi^*)dx$$

For the operator xp, we have

$$\begin{aligned}
\int \psi^*(xp)^\dagger \psi dx &= \int \psi(xp)^* \psi^* dx \\
&= (-i\hbar)^* \int \psi x \frac{\partial \psi^*}{\partial x} dx \\
&= -i\hbar \int \frac{\partial x\psi}{\partial x} \psi^* dx \\
&= \int \psi^*(px)\psi dx
\end{aligned}$$

So we have, as desired

$$(xp)^\dagger = px$$

1.12 P12. Hermitian operator from antisymmetric difference.

Show that $H = i(O - O^\dagger)$ is a Hermitian operator.

This follows directly from the definition, calculating the expectation

$$\begin{aligned}
\langle H \rangle &= \int \psi^*(i(O - O^\dagger)\psi) \\
&= i \int \psi^*(O\psi) - i \int \psi^*(O^\dagger \psi) \\
&= i \int \psi^*(O\psi) - i \int \psi(O^* \psi^*)
\end{aligned}$$

Taking conjugates we have

$$\langle H \rangle^* = -i \int \psi(O^*\psi^*) + i \int \psi^*(O\psi)$$
$$= \langle H \rangle$$
$$\square$$

1.13   P13. When product of operators is Hermitian.

What relation must exist between Hermitian B and A must exist for AB to be Hermitian.

TODO: Am guessing that this has something to do with the commutator of the operators. This one I don't have a check mark besides in my text, so did I ever figure it out?

1.14   P14. Show directly that $i(p^2x - xp^2)$ is Hermitian

This follows from $(AB)^\dagger = BA$ in the text above. We have

$$(i(p^2x - xp^2))^\dagger = (x^\dagger p^\dagger p^\dagger - p^\dagger p^\dagger x^\dagger)(-i)$$
$$= -i(xp^2 - p^2x)$$
$$= i(p^2x - xp^2) \qquad \square$$

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