DC Power consumption formula for resistive load.

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1 Motivation.

Despite a lot of recent study of electrodynamics, faced with a simple electrical problem:

"What capacity generator would be required for an arc welder on a 30 Amp breaker using a 220 volt circuit".

I couldn't think of how to answer this off the top of my head. Back in school without hesitation I would have been able to plug into $P = IV$ to get a capacity estimation for the generator.

Having forgotten the formula, let's see how we get that $P = IV$ relationship from Maxwell's equations.

2

Having just derived the Poynting energy momentum density relationship from Maxwell's equations, let that be the starting point

$$\frac{d}{dt}\left(\frac{\epsilon_0}{2}\left(\mathbf{E}^2 + c^2\mathbf{B}^2\right)\right) = -\frac{1}{\mu_0}\left(\mathbf{E} \times \mathbf{B}\right) - \mathbf{E} \cdot \mathbf{j}$$

The left hand side is the energy density time variation, which is power per unit volume, so we can integrate this over a volume to determine the power associated with a change in the field.

$$P = -\int dV \left(\frac{1}{\mu_0}\left(\mathbf{E} \times \mathbf{B}\right) + \mathbf{E} \cdot \mathbf{j}\right)$$

As a reminder, let's write the magnetic and electric fields in terms of potentials.

In terms of the "native" four potential our field is

$$
\begin{aligned}
F &= \mathbf{E} + ic\mathbf{B} \\
&= \nabla \wedge A \\
&= \gamma^0 \gamma_k \partial_0 A^k + \gamma^j \gamma_0 \partial_j A^0 + \gamma^m \wedge \gamma_n \partial_m A^n
\end{aligned}
$$

The electric field is

$$
\mathbf{E} = \sum_k (\nabla \wedge A) \cdot (\gamma^0 \gamma^k) \gamma_k \gamma_0
$$

From this, with $\phi = A^0$, and $\mathbf{A} = \sigma_k A^k$ we have

$$
\begin{aligned}
\mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \\
i\mathbf{B} &= \boldsymbol{\nabla} \wedge \mathbf{A}
\end{aligned}
$$

Now, the arc welder is (I think) a DC device, and to get a rough idea of what it requires lets just assume that its a rectifier that outputs RMS DC. So if we make this simplification, and assume that we have a purely resistive load (ie: no inductance and therefore no magnetic fields) and a DC supply and constant current, then we eliminate the vector potential terms.

This wipes out the $\mathbf{B}$ and the Poynting vector, and leaves our electric field specified in terms of the potential difference accross the load $\mathbf{E} = -\boldsymbol{\nabla}\phi$.

That is

$$
P = \int dV (\boldsymbol{\nabla}\phi) \cdot \mathbf{j}
$$

Suppose we are integrating over the length of a uniformly resistive load with some fixed cross sectional area. $\mathbf{j} dV$ is then the magnitude of the current directed along the wire for its length. This basically leaves us with a line integral over the length of the wire that we are measuring our potential drop over so we are left with just

$$
P = (\delta\phi) I
$$

This $\delta\phi$ is just our voltage drop V, and this gives us the desired $P = IV$ equation. Now, I also recall from school now that I think about it that $P = IV$ also applied to inductive loads, but it required that I and V be phasors that represented the sinusodal currents and sources. A good followup exersize would be to show from Maxwell's equations that this is in fact valid. Eventually I'll know the origin of all the formulas from my old engineering courses.