polar velocity and accerlation.

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Jan 13, 2009. Last Revision: $Date : 2009/01/1323 : 25 : 03$

1 Motivation.

Have previously worked out the radial velocity and acceleration components a pile of different ways in [Joot(a)], [Joot(b)], [Joot(c)], [Joot(d)], [Joot(e)], and [Joot(f)].

So, what's a couple more?

When the motion is strictly restricted to a plane we can get away with doing this either in complex numbers (used in a number of the Tong Lagrangian solutions), or with a polar form $\mathbb{R}^2$ vector (a polar representation I haven't seen since High School).

2 With complex numbers.

Let

$$z = re^{i\theta}$$

So our velocity is

$$\dot{z} = \dot{r}e^{i\theta} + ir\dot{\theta}e^{i\theta}$$

and the acceleration is

$$\begin{aligned}\ddot{z} &= \ddot{r}e^{i\theta} + i\dot{r}\dot{\theta}e^{i\theta} + i\dot{r}\dot{\theta}e^{i\theta} + ir\ddot{\theta}e^{i\theta} - r\dot{\theta}^2e^{i\theta} \\ &= (\ddot{r} - r\dot{\theta}^2)e^{i\theta} + (2\dot{r}\dot{\theta} + r\ddot{\theta})ie^{i\theta}\end{aligned}$$

3 Plane vector representation.

Also can do this with polar vector representation directly (without involving the complexity of rotation matrixes or anything fancy)

$$\mathbf{r} = r \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

Velocity is then

$$\mathbf{v} = \dot{r} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + r\dot{\theta} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

and for acceleration we have

$$\begin{aligned} \mathbf{a} &= \ddot{r} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \dot{r}\dot{\theta} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} + \dot{r}\dot{\theta} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} + r\ddot{\theta} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} - r\dot{\theta}^2 \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \\ &= (\ddot{r} - r\dot{\theta}^2) \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \end{aligned}$$

References

[Joot(a)] Peeter Joot. Angular acceleration. "http://sites.google.com/site/peeterjoot/geometric-algebra/angular_acc.pdf", a.

[Joot(b)] Peeter Joot. Angular acceleration using cross product . "http://sites.google.com/site/peeterjoot/geometric-algebra/angular_acc_cross.pdf", b.

[Joot(c)] Peeter Joot. Angular velocity. "http://sites.google.com/site/peeterjoot/geometric-algebra/angular_velocity.pdf", c.

[Joot(d)] Peeter Joot. Rotational kinetic energy. "http://sites.google.com/site/peeterjoot/geometric-algebra/ke_rotation.pdf", d.

[Joot(e)] Peeter Joot. Radial vector derivatives. "http://sites.google.com/site/peeterjoot/geometric-algebra/radial_vector_derivatives.pdf", e.

[Joot(f)] Peeter Joot. Unit vector derivatives. "http://sites.google.com/site/peeterjoot/geometric-algebra/ga_wiki_unit_derivative.pdf", f.