PHY450H1S. Relativistic Electrodynamics Lecture 5 (Taught by Prof. Erich Poppitz). Spacetime, events, worldlines, spacetime intervals, and invariance.

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http://sites.google.com/site/peeterjoot/math2011/relativisticElectrodynamicsL5.pdf

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1. Reading.

Still covering chapter 1 material from the text [1]?

Covering Professor Poppitz's lecture notes: Using Minkowski diagram to see the perils of superluminal propagation (32.3); nonrelativistic limit of boosts (33); number of parameters of Lorentz transformations (34-35); introducing four-vectors, the metric tensor, the invariant "dot-product and SO(1,3) (36-40); the Poincare group (41); the convenience of "upper" and "lower" indices (42-43); tensors (44)

2. More on proper time.

PICTURE:1: worldline with small interval.

Considering a small interval somewhere on the worldline trajectory, we have

$$ds^2 = c^2 dt^2 - dx^2 = c^2 {dt'}^2, \tag{1}$$

where dt' is the proper time elapsed in a frame moving with velocity v, and dt is the time elapsed in a stationary frame.

We have

$$dt' = dt \sqrt{1 - (dx/dt)^2/c^2} = dt \sqrt{1 - v^2/c^2}. \tag{2}$$

PICTURE:2: particle at rest.

For the particle at rest

$$c\tau_{21}^{\text{stationary}} = c(t_2 - t_1) = \int_1^2 ds = \int_1^2 cdt \tag{3}$$

PICTURE:3: particle with motion.

"length" of 1-2 "curved" worldline

$$\begin{aligned}\int_1^2 ds' &= \int_1^2 cdt' \\ &= \int_1^2 cdt \sqrt{1 - (d\mathbf{v}/dt)^2},\end{aligned}$$

where in this case $[1,2]$ denotes the range of a line integral over the worldline. We see that the multiplier of dt for any point along the curve is smaller than 1, so that the length along a straight line is longest (i.e. for the particle at rest).

We've argued that if 1,2 occur at the same place, the spacetime length of a straight line between them is the longest. This remains the time for all 1,2 timelike separated.

LOTS OF DISCUSSION. See new posted notes for details.

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We've argued that $ds_{12} = ds'_{12} \implies s_{12} = s'_{12}$ for infinitesimal 1,2 even if not infinitesimal.

The idea is to represent the interval between two not close 1,2 as a sum over small ds's.

P6: $x = x_2 t / t_2$ straight line through origin, with $t \in [0, t_2]$.

P7: zoomed on part of this line.

$$\begin{aligned} ds^2 &= c^2 dt^2 - dx^2 \\ &= c^2 dt^2 - \left(\frac{x_2}{t_2}\right)^2 dt^2 \\ &= c^2 dt^2 \left(1 - \frac{1}{c^2}\left(\frac{x_2}{t_2}\right)^2\right) \end{aligned}$$

or

$$\int_0^1 ds = c \int_0^{t_2} dt \sqrt{1 - \frac{1}{c^2}\left(\frac{x_2}{t_2}\right)^2} \tag{4}$$

In another frame just replace $t \to t'$ and $x_2 \to x'_2$

$$\int_0^1 ds = c \int_0^{t'_2} dt \sqrt{1 - \frac{1}{c^2}\left(\frac{x'_2}{t'_2}\right)^2} \tag{5}$$

3. **Length contraction.**

Consider O and O' with O' moving in x with speed $v_x > 0$. Here we have

$$x' = \gamma\left(x - \frac{v_x}{c} ct\right) \tag{6}$$

$$ct' = \gamma\left(ct - \frac{v_x}{c} x\right) \tag{7}$$

PICTURE: spacetime diagram with ct' at angle α, where $\tan\alpha = v_x/c$.

Two points $(x_A, 0)$, $(x_B, 0)$, with rest length measured as $L = x_B - x_A$. From the diagram $c(t_B - t_A) = \tan\alpha L$, and from 6 we have

$$x'_A = \gamma\left(x_A - \frac{v_x}{c} ct_A\right) \tag{8}$$

$$x'_B = \gamma\left(x_B - \frac{v_x}{c} ct_B\right), \tag{9}$$

so that

$$
\begin{aligned}
L' &= x'_B - x'_A \\
&= \gamma\left((x_B - x_A) - \frac{v_x}{c}c(t_B - t_A)\right) \\
&= \gamma\left(L - \frac{v_x}{c}\tan\alpha L\right) \\
&= \gamma\left(L - \frac{v_x^2}{c^2}L\right) \\
&= \gamma L\left(1 - \frac{v_x^2}{c^2}\right) \\
&= L\sqrt{1 - \frac{v_x^2}{c^2}}
\end{aligned}
$$

4. **Superluminal speed and causality.**

If Einstein's relativity holds, superliminal motion is a "no-no". Imagine that some "tachyons" exist that can instantaneously transmit stuff between observers.

PICTURE9: two guys with resting worldlines showing.

Can send info back to A before A sends to B. Superluminal propagation allows sending information not yet available. Can show this for finite superluminal velocities (but hard) as well as infinite velocity superluminal speeds. We see that time ordering can not be changed for events separated by time like separation. Events separated by spacelike separation cannot be ca usually connected.

References

[1] L.D. Landau and E.M. Lifshits. *The classical theory of fields*. Butterworth-Heinemann, 1980. 1