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Laplace transform refresher

Laplace transforms were used to solve the MNA equations for time dependent systems, and to find the moments used to in MOR.

For the record, the Laplace transform is defined as:

$$\boxed{\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t) dt.} \tag{1.1}$$

The only Laplace transform pair used in the lectures is that of the first derivative

$$\begin{aligned}\mathcal{L}(f'(t)) &= \int_0^\infty e^{-st} \frac{df(t)}{dt} dt \\ &= e^{-st} f(t)\Big|_0^\infty - (-s) \int_0^\infty e^{-st} f(t) dt \\ &= -f(0) + s\mathcal{L}(f(t)).\end{aligned} \tag{1.2}$$

Here it is loosely assumed that the real part of s is positive, and that $f(t)$ is "well defined" enough that $e^{-s\infty} f(\infty) \rightarrow 0$.

Where used in the lectures, the laplace transforms were of vectors such as the matrix vector product $\mathcal{L}(\mathbf{G}\mathbf{x}(t))$. Because such a product is linear, observe that it can be expressed as the original matrix times a vector of Laplace transforms

$$\begin{aligned}\mathcal{L}(\mathbf{G}\mathbf{x}(t)) &= \mathcal{L}\left[G_{ik} x_k(t) \right]_i \\ &= \left[G_{ik} \mathcal{L} x_k(t) \right]_i \\ &= \mathbf{G} \left[\mathcal{L} x_i(t) \right]_i.\end{aligned} \tag{1.3}$$

The following notation was used in the lectures for such a vector of Laplace transforms

$$\mathbf{X}(s) = \mathcal{L}\mathbf{x}(t) = \left[\mathcal{L} x_i(t) \right]_i. \tag{1.4}$$