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Can anticommuting operators have a simultaneous eigenket?

Exercise 1.1 **Can anticommuting operators have a simultaneous eigenket? (*[1] pr. 1.16*)**

Two Hermitian operators anticommute

$$\begin{aligned}\{A, B\} &= AB + BA \\ &= 0.\end{aligned} \tag{1.1}$$

Is it possible to have a simultaneous eigenket of A and B? Prove or illustrate your assertion.

Answer for Exercise 1.1

Suppose that such a simultaneous non-zero eigenket $|\alpha\rangle$ exists, then

$$A |\alpha\rangle = a |\alpha\rangle, \tag{1.2}$$

and

$$B |\alpha\rangle = b |\alpha\rangle \tag{1.3}$$

This gives

$$\begin{aligned}(AB + BA) |\alpha\rangle &= (Ab + Ba) |\alpha\rangle \\ &= 2ab |\alpha\rangle.\end{aligned} \tag{1.4}$$

If this is zero, one of the operators must have a zero eigenvalue. Knowing that we can construct an example of such operators. In matrix form, let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & a \end{bmatrix} \tag{1.5a}$$

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & b \end{bmatrix}. \tag{1.5b}$$

These are both Hermitian, and anticommute provided at least one of a, b is zero. These have a common eigenket

$$|\alpha\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{1.6}$$

A zero eigenvalue of one of the commuting operators may not be a sufficient condition for such anticommutation.

Bibliography

[1] Jun John Sakurai and Jim J Napolitano. *Modern quantum mechanics*. Pearson Higher Ed, 2014. 1.1