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Correlation function

Exercise 1.1 Correlation function ([1] *pr. 2.16*)

A correlation function can be defined as

$$C(t) = \langle x(t)x(0) \rangle . \tag{1.1}$$

Using a Heisenberg picture $x(t)$ calculate this correlation for the one dimensional SHO ground state.

Answer for Exercise 1.1

The time dependent Heisenberg picture position operator was found to be

$$x(t) = x(0)\cos(\omega t) + \frac{p(0)}{m\omega}\sin(\omega t), \tag{1.2}$$

so the correlation function is

$$\begin{aligned} C(t) &= \langle 0 | \left(x(0)\cos(\omega t) + \frac{p(0)}{m\omega}\sin(\omega t) \right) x(0) | 0 \rangle \\ &= \cos(\omega t) \langle 0 | x(0)^2 | 0 \rangle + \frac{\sin(\omega t)}{m\omega} \langle 0 | p(0)x(0) | 0 \rangle \\ &= \frac{\hbar\cos(\omega t)}{2m\omega} \langle 0 | \left(a + a^\dagger \right)^2 | 0 \rangle - \frac{i\hbar}{m\omega}\sin(\omega t), \end{aligned} \tag{1.3}$$

But

$$\begin{aligned} \left(a + a^\dagger \right) | 0 \rangle &= a^\dagger | 0 \rangle \\ &= \sqrt{1} | 1 \rangle \\ &= | 1 \rangle , \end{aligned} \tag{1.4}$$

so

$$C(t) = x_0^2 \left(\frac{1}{2}\cos(\omega t) - i\sin(\omega t) \right) , \tag{1.5}$$

where $x_0^2 = \hbar/(m\omega)$, not to be confused with $x(0)^2$.

Bibliography

[1] Jun John Sakurai and Jim J Napolitano. *Modern quantum mechanics*. Pearson Higher Ed, 2014. 1.1