Duality transformation of the far field fields.

We’ve seen that the far field electric and magnetic fields associated with a magnetic vector potential were

\[ E = -j\omega \text{Proj}_T A, \]
\[ H = \frac{1}{\eta} \hat{k} \times E. \]  

(1.1a)  
(1.1b)

It’s worth a quick note that the duality transformation for this, referring to [1] tab.3.2, is

\[ H = -j\omega \text{Proj}_T F \]
\[ E = -\eta \hat{k} \times H. \]  

(1.2a)  
(1.2b)

What does \( H \) look like in terms of \( A \), and \( E \) look like in terms of \( H \)?

The first is

\[ H = -\frac{j\omega}{\eta} \hat{k} \times \left( A - \left( A \cdot \hat{k} \right) \hat{k} \right), \]  

(1.3)

in which the \( \hat{k} \) crossed terms are killed, leaving

\[ H = -\frac{j\omega}{\eta} \hat{k} \times A. \]  

(1.4)

The electric field follows again using a duality transformation, so in terms of the electric vector potential, is

\[ E = j\omega \eta \hat{k} \times F. \]  

(1.5)

These show explicitly that neither the electric or magnetic far field have any radial component, matching with intuition for transverse propagation of the fields.
Bibliography