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Duality transformation of the far field fields.

We've seen that the far field electric and magnetic fields associated with a magnetic vector potential were

$$\mathbf{E} = -j\omega \operatorname{Proj}_{\mathrm{T}} \mathbf{A}, \tag{1.1a}$$

$$\mathbf{H} = \frac{1}{\eta} \hat{\mathbf{k}} \times \mathbf{E}. \tag{1.1b}$$

It's worth a quick note that the duality transformation for this, referring to [1] tab.3.2, is

$$\mathbf{H} = -j\omega \operatorname{Proj}_{\mathrm{T}} \mathbf{F} \tag{1.2a}$$

$$\mathbf{E} = -\eta \hat{\mathbf{k}} \times \mathbf{H}. \tag{1.2b}$$

What does $\mathbf{H}$ look like in terms of $\mathbf{A}$, and $\mathbf{E}$ look like in terms of $\mathbf{H}$?
The first is

$$\mathbf{H} = -\frac{j\omega}{\eta} \hat{\mathbf{k}} \times \left(\mathbf{A} - \left(\mathbf{A} \cdot \hat{\mathbf{k}} \right) \hat{\mathbf{k}} \right), \tag{1.3}$$

in which the $\hat{\mathbf{k}}$ crossed terms are killed, leaving

$$\mathbf{H} = -\frac{j\omega}{\eta} \hat{\mathbf{k}} \times \mathbf{A}. \tag{1.4}$$

The electric field follows again using a duality transformation, so in terms of the electric vector potential, is

$$\mathbf{E} = j\omega\eta \hat{\mathbf{k}} \times \mathbf{F}. \tag{1.5}$$

These show explicitly that neither the electric or magnetic far field have any radial component, matching with intuition for transverse propagation of the fields.

Bibliography

[1] Constantine A Balanis. *Antenna theory: analysis and design*. John Wiley & Sons, 3rd edition, 2005.