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Impedance transformation

In our final problem set we used the impedance transformation for calculations related to a microslot antenna. This transformation wasn't familiar to me, and is apparently covered in the third year ECE fields class. I found a derivation of this in [1], but the idea is really simple and follows from the reflection coefficient calculation for a normal reflection configuration.

Consider a normal field reflection between two interfaces, as sketched in fig. 1.1.

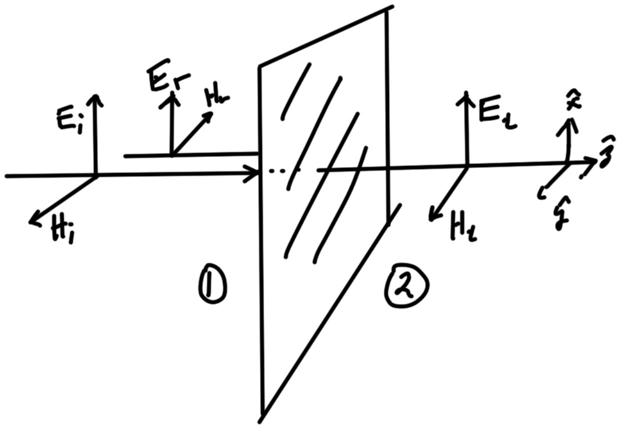

Figure 1.1: Normal reflection and transmission between two media.

The fields are

$$\mathbf{E}^{\mathrm{i}} = \hat{\mathbf{x}} E_0 e^{-jk_1 z} \tag{1.1a}$$

$$\mathbf{H}^{\mathrm{i}} = \hat{\mathbf{y}} \frac{E_0}{\eta_1} e^{-jk_1 z} \tag{1.1b}$$

$$\mathbf{E}^{\mathrm{r}} = \hat{\mathbf{x}} \Gamma E_0 e^{jk_1 z} \tag{1.1c}$$

$$\mathbf{H}^{\mathrm{r}} = -\hat{\mathbf{y}} \Gamma \frac{E_0}{\eta_1} e^{jk_1 z} \tag{1.1d}$$

$$\mathbf{E}^{\mathrm{t}} = \hat{\mathbf{x}} E_0 T e^{-jk_2 z} \tag{1.1e}$$

$$\mathbf{H}^{\mathrm{t}} = \hat{\mathbf{y}} \frac{E_0}{\eta_1} T e^{-jk_2 z}. \tag{1.1f}$$

The field orientations have been picked so that the tangential component of the electric field is $\hat{\mathbf{x}}$ oriented for all of the incident, reflected, and transmitted components. Requiring equality of the tangential field components at the interface gives

$$1 + \Gamma = T \tag{1.2a}$$

$$\frac{1}{\eta_1} - \frac{\Gamma}{\eta_1} = \frac{T}{\eta_2}. \tag{1.2b}$$

Solving for the transmission coefficient gives

$$\begin{aligned} T &= \frac{2}{1 + \frac{\eta_1}{\eta_2}} \\ &= \frac{2\eta_2}{\eta_2 + \eta_1}, \end{aligned} \tag{1.3}$$

and for the reflection coefficient

$$\begin{aligned} \Gamma &= T - 1 \\ &= \frac{2\eta_2 - \eta_1 - \eta_2}{\eta_2 + \eta_1} \\ &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}. \end{aligned} \tag{1.4}$$

The total fields in medium 1 at the point $z = -l$ are

$$\mathbf{E}^{\mathrm{i}} + \mathbf{E}^{\mathrm{r}} = \hat{\mathbf{x}} E_0 \left(e^{-jk_1(-l)} + \Gamma e^{jk_1(-l)} \right) \tag{1.5a}$$

$$\mathbf{H}^{\mathrm{i}} + \mathbf{H}^{\mathrm{r}} = \hat{\mathbf{y}} \frac{E_0}{\eta_1} \left(e^{-jk_1(-l)} - \Gamma e^{jk_1(-l)} \right). \tag{1.5b}$$

The ratio of the electric field strength to the magnetic field strength is defined as the input impedance

$$Z_{\mathrm{in}} \equiv \left. \frac{E^{\mathrm{i}} + E^{\mathrm{r}}}{H^{\mathrm{i}} + H^{\mathrm{r}}} \right|_{z=-l}. \tag{1.6}$$

That is

$$
\begin{aligned}
Z_{\text{in}} &= \eta_1 \frac{e^{jk_1 l} + \Gamma e^{-jk_1 l}}{e^{jk_1 l} - \Gamma e^{-jk_1 l}} \\
&= \eta_1 \frac{\left(\eta_1 + \eta_2\right) e^{jk_1 l} + \left(\eta_2 - \eta_1\right) e^{-jk_1 l}}{\left(\eta_1 + \eta_2\right) e^{jk_1 l} - \left(\eta_2 - \eta_1\right) e^{-jk_1 l}} \\
&= \eta_1 \frac{\eta_2 \cos(k_1 l) + \eta_1 j \sin(k_1 l)}{\eta_2 j \sin(k_1 l) + \eta_1 \cos(k_1 l)},
\end{aligned} \tag{1.7}
$$

or

$$
\boxed{Z_{\text{in}} = \eta_1 \frac{\eta_2 + j\eta_1 \tan(k_1 l)}{\eta_1 + j\eta_2 \tan(k_1 l)}.} \tag{1.8}
$$

Bibliography

[1] Constantine A Balanis. *Advanced engineering electromagnetics*, chapter Reflection and transmission. Wiley New York, 1989. 1