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Quadratic Zeeman effect

Q: [1] pr. 5.18 Work out the quadratic Zeeman effect for the ground state hydrogen atom due to the usually neglected $e^2\mathbf{A}^2/2m_e c^2$ term in the Hamiltonian.

A: The first order energy shift is
For a z-oriented magnetic field we can use

$$\mathbf{A} = \frac{B}{2}\{-y, x, 0\}, \tag{1.1}$$

so the perturbation potential is

$$\begin{aligned} V &= \frac{e^2\mathbf{A}^2}{2m_e c^2} \\ &= \frac{e^2\mathbf{B}^2(x^2+y^2)}{8m_e c^2} \\ &= \frac{e^2\mathbf{B}^2 r^2 \sin^2\theta}{8m_e c^2} \end{aligned} \tag{1.2}$$

The ground state wave function is

$$\begin{aligned} \psi_0 &= \langle \mathbf{x} | 0 \rangle \\ &= \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \end{aligned} \tag{1.3}$$

so the energy shift is

$$
\begin{aligned}
\Delta &= \langle 0| V |0\rangle \\
&= \frac{1}{\pi a_0^3} 2\pi \frac{e^2 \mathbf{B}^2}{8 m_e c^2} \int_0^\infty r^2 \sin\theta e^{-2r/a_0} r^2 \sin^2\theta dr d\theta \\
&= \frac{e^2 \mathbf{B}^2}{4 a_0^3 m_e c^2} \int_0^\infty r^4 e^{-2r/a_0} dr \int_0^\pi \sin^3\theta d\theta \\
&= -\frac{e^2 \mathbf{B}^2}{4 a_0^3 m_e c^2} \frac{4!}{(2/a_0)^{4+1}} \left(u - \frac{u^3}{3} \right)\Bigg|_1^{-1} \\
&= \frac{e^2 a_0^2 \mathbf{B}^2}{4 m_e c^2}.
\end{aligned} \tag{1.4}
$$

If this energy shift is written in terms of a diamagnetic susceptibility χ defined by

$$
\Delta = -\frac{1}{2}\chi \mathbf{B}^2, \tag{1.5}
$$

the diamagnetic susceptibility is

$$
\chi = -\frac{e^2 a_0^2 \mathbf{B}^2}{2 m_e c^2}. \tag{1.6}
$$

Bibliography

[1] Jun John Sakurai and Jim J Napolitano. *Modern quantum mechanics*. Pearson Higher Ed, 2014. 1