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Scalar field creation operator commutator

Exercise 1.1

In [1] it is stated that the creation operators of eq. 2.78

$$\alpha_k = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \left(\phi(x, 0) + \frac{i}{\omega_k} \partial_0 \phi(x, 0) \right) e^{-i \mathbf{k} \cdot \mathbf{x}} \tag{1.1}$$

associated with field operator ϕ commute. Verify that.

Answer for Exercise 1.1

$$\begin{aligned}
[\alpha_k, \alpha_m] &= \frac{1}{4} \frac{1}{(2\pi)^6} \int d^3 x d^3 y e^{-i \mathbf{k} \cdot \mathbf{x}} e^{-i \mathbf{m} \cdot \mathbf{y}} \left[\phi(x, 0) + \frac{i}{\omega_k} \partial_0 \phi(x, 0), \phi(y, 0) + \frac{i}{\omega_m} \partial_0 \phi(y, 0) \right] \\
&= \frac{i}{4} \frac{1}{(2\pi)^6} \int d^3 x d^3 y e^{-i \mathbf{k} \cdot \mathbf{x}} e^{-i \mathbf{m} \cdot \mathbf{y}} \left(\left[\phi(x, 0), \frac{1}{\omega_m} \partial_0 \phi(y, 0) \right] + \left[\frac{1}{\omega_k} \partial_0 \phi(x, 0), \phi(y, 0) \right] \right) \\
&= \frac{i}{4} \frac{1}{(2\pi)^6} \int d^3 x d^3 y e^{-i \mathbf{k} \cdot \mathbf{x}} e^{-i \mathbf{m} \cdot \mathbf{y}} \left(\frac{i}{\omega_m} \delta^3(\mathbf{x} - \mathbf{y}) - \frac{i}{\omega_k} \delta^3(\mathbf{x} - \mathbf{y}) \right) \\
&= -\frac{1}{4} \frac{1}{(2\pi)^6} \int d^3 x e^{-i(\mathbf{k} + \mathbf{m}) \cdot \mathbf{x}} \left(\frac{1}{\omega_m} - \frac{1}{\omega_k} \right) \\
&= -\frac{1}{4} \frac{1}{(2\pi)^3} \left(\frac{1}{\omega_m} - \frac{1}{\omega_k} \right) \delta^3(\mathbf{k} + \mathbf{m}) \\
&= -\frac{1}{4} \frac{1}{(2\pi)^3} \left(\frac{1}{\omega_{|-\mathbf{k}|}} - \frac{1}{\omega_{|\mathbf{k}|}} \right) \delta^3(\mathbf{k} + \mathbf{m}) \\
&= 0.
\end{aligned} \tag{1.2}$$

Bibliography

[1] Michael Luke. *PHY2403F Lecture Notes: Quantum Field Theory*, 2015. URL https://s3.amazonaws.com/piazza-resources/i87nj8g7yie7nh/ihdwuk7wva13qq/lecturenotes.pdf?AWSAccessKeyId=AKIAIEDNRLJ4AZKBW6HA&Expires=1451803428&Signature=IF6qOjlKqOYL01FwqT%2FGV6BSDb8%3D. [Online; accessed 02-Jan-2016]. 1.1