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Exercise 2.1 Time reversal, crystal spin Hamiltonian. ([1] *pr. 4.12*)

Solve the spin 1 Hamiltonian

$$H = A S_z^2 + B(S_x^2 - S_y^2). \tag{2.1}$$

Is this Hamiltonian invariant under time reversal? How do the eigenkets change under time reversal?

Answer for Exercise 2.1

In spinMatrices.nb the matrix representation of the Hamiltonian is found to be

$$H = \hbar^2 \begin{bmatrix} A & 0 & B \\ 0 & 0 & 0 \\ B & 0 & A \end{bmatrix}. \tag{2.2}$$

The eigenvalues are

$$\left\{ 0, A - B, A + B \right\}, \tag{2.3}$$

and the respective eigenvalues (unnormalized) are

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}. \tag{2.4}$$

Under time reversal, the Hamiltonian is

$$H \rightarrow A(-S_z)^2 + B((-S_x)^2 - (-S_y)^2) = H, \tag{2.5}$$

so we expect the eigenkets for this Hamiltonian to vary by at most a phase factor. To check this, first recall that the time reversal action on a spin one state is

$$\Theta \left| 1, m \right\rangle = (-1)^m \left| 1, -m \right\rangle, \tag{2.6}$$

or

$$\begin{aligned}
\Theta \left| 1, 1 \right\rangle &= - \left| 1, -1 \right\rangle \\
\Theta \left| 1, 0 \right\rangle &= \left| 1, 0 \right\rangle \\
\Theta \left| 1, -1 \right\rangle &= - \left| 1, 1 \right\rangle .
\end{aligned} \tag{2.7}$$

Let's write the eigenkets respectively as

$$\begin{aligned}
\left| 0 \right\rangle &= \left| 1, 0 \right\rangle \\
\left| A - B \right\rangle &= - \left| 1, -1 \right\rangle + \left| 1, 1 \right\rangle \\
\left| A + B \right\rangle &= \left| 1, -1 \right\rangle + \left| 1, 1 \right\rangle .
\end{aligned} \tag{2.8}$$

Under the reversal operation, we should have

$$\begin{aligned}
\Theta \left| 0 \right\rangle &\rightarrow \left| 1, 0 \right\rangle \\
\Theta \left| A - B \right\rangle &= + \left| 1, -1 \right\rangle - \left| 1, 1 \right\rangle \\
\Theta \left| A + B \right\rangle &= - \left| 1, -1 \right\rangle - \left| 1, 1 \right\rangle .
\end{aligned} \tag{2.9}$$

Up to a sign, the time reversed states match the unreversed states.

Bibliography

[1] Jun John Sakurai and Jim J Napolitano. *Modern quantum mechanics*. Pearson Higher Ed, 2014. 2.1