Errata for Feynman's Quantum Electrodynamics (Addison-Wesley)?

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1. Motivation.

I got a nice present today which included one of Feynman's QED books (Addison-Wesley Feb 98 first printing). I noticed some early mistakes, and since I can't find an errata page anywhere, I'll collect them here, along with some other notes.

Eventually, if I get through the book, I'll see about sending this into the publisher.

2. On what I believe should be in the errata if it existed.

2.1. *Third Lecture*

2.1.1 Page 6.

The electric field is given in terms of only the scalar potential

$$\mathbf{E} = -\boldsymbol{\nabla}\phi + \partial\phi/\partial t,$$

and should be

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \frac{1}{c}\partial \mathbf{A}/\partial t.$$

The invariant gauge transformation for the vector and scalar potentials are given as

$$\begin{aligned}
\mathbf{A}' &= \mathbf{A} + \boldsymbol{\nabla}\chi \\
\phi' &= \phi + \partial\chi/\partial t
\end{aligned}$$

But these should be

$$\begin{aligned}
\mathbf{A}' &= \mathbf{A} + \boldsymbol{\nabla}\chi \\
\phi' &= \phi - \frac{1}{c}\partial\chi/\partial t
\end{aligned}$$

The sign was crossed on the scalar potential transformation. Perhaps Feynman used $c = 1$ in his lectures, and whoever made the notes wasn't consistent about including these in all the right places (but did so in some).

2.1.2 Page 7.

With the signs and constant terms of the gauge transformation for the potentials being off, so is the end result for the final set of transformations that leave the Pauli equation invariant. That should be:

$$\begin{aligned}
\mathbf{A}' &= \mathbf{A} + \boldsymbol{\nabla}\chi \\
\phi' &= \phi - \frac{1}{c}\frac{\partial\chi}{\partial t} \\
\Psi' &= \exp\left(i\frac{e}{\hbar c}\chi\right)\Psi,
\end{aligned}$$

(with the intermediate steps corrected accordingly).

2.1.3 Page 8.

It's written

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 - \frac{e\hbar}{2mc}(\sigma \cdot \boldsymbol{\nabla} \times \mathbf{A}) + eV$$

It appears that the minus should be a positive here.

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + \frac{e\hbar}{2mc}(\sigma \cdot \boldsymbol{\nabla} \times \mathbf{A}) + eV$$

It also appears that $\mathbf{x}^2 \equiv (\mathbf{x}\cdot\mathbf{x})I$, where the identity matrix I is implied.
Then, equation (1) which reads

$$\nabla \times \mathbf{A} = \mathbf{K} \times \mathbf{e} e^{i\mathbf{K}\cdot\mathbf{x}} e^{i\omega t}$$

should be

$$\begin{aligned}
\nabla \times \mathbf{A} &= a\mathbf{K} \times \mathbf{e} e^{i\mathbf{K}\cdot\mathbf{x}} e^{-i\omega t} - a e^{i\mathbf{K}\cdot\mathbf{x}} e^{-i\omega t} \mathbf{e} \times \nabla \\
&= \mathbf{A} \times (i\mathbf{K} - \nabla)
\end{aligned}$$

And in equation two the sign is wrong. It reads

$$\mathbf{p} e^{i\mathbf{K}\cdot\mathbf{x}} = e^{i\mathbf{K}\cdot\mathbf{x}} (\mathbf{p} - \hbar\mathbf{K})$$

but should be

$$\mathbf{p} e^{i\mathbf{K}\cdot\mathbf{x}} = e^{i\mathbf{K}\cdot\mathbf{x}} (\mathbf{p} + \hbar\mathbf{K})$$

similarly the following $-\hbar\mathbf{K} \cdot \mathbf{e}$ should be positive, $\hbar\mathbf{K} \cdot \mathbf{e}$. (this last has no effect since $\mathbf{K} \cdot \mathbf{e}$ is assumed zero since $\mathbf{e}$ was picked as the transverse propagation direction for the electrodynamic wave).

2.2. *Seventh Lecture*

2.2.1 Page 25.

Last equation reads

$$Et - p_x x - p_y y - p_z z = p_\mu p_\mu$$

should be

$$Et - p_x x - p_y y - p_z z = p_\mu x_\mu$$

2.2.2 Page 26.

After "but" we have

$$p_0^2 = E^2 - m$$

which should be

$$p_0^2 = E^2 - m^2$$

2.2.3 Page 29.

The gauge transformation once again has the sign messed up. It was written (from ${A_\mu}' = A_\mu + \nabla_\mu \chi$)

$$\begin{aligned}\mathbf{A}' &= \mathbf{A} + \boldsymbol{\nabla}\chi \\ \phi' &= \phi + \partial\chi/\partial t\end{aligned}$$

but it should be

$$\begin{aligned}\mathbf{A}' &= \mathbf{A} - \boldsymbol{\nabla}\chi \\ \phi' &= \phi + \partial\chi/\partial t\end{aligned}$$

(ie: $\nabla_m = -\partial_m$)
Then a bit later

$$\nabla \cdot A' = \nabla \cdot A + \nabla \cdot \chi$$

should be

$$\nabla \cdot A' = \nabla \cdot A + \nabla \cdot \nabla\chi$$

2.2.4 Page 29.

$$dx/ds = (dx/dt)(dt/ds) = v_x/(1-y^2)^{1/2}$$

should be

$$dx/ds = (dx/dt)(dt/ds) = v_x/(1-v^2)^{1/2}$$

3. Extended notes.

3.1. *Second Lecture*

This isn't errata, but I found the following required slight exploration. He gives (implicitly)

$$\overline{\sin^2(\omega t - \mathbf{K} \cdot \mathbf{x})} = \frac{1}{2}$$

Is this an average over space and time? How would one do that? What do we get just integrating this over the volume? That dot product is $\mathbf{K} \cdot \mathbf{x} = 2\pi \left(\frac{m}{\lambda_1} x + \frac{n}{\lambda_2} y + \frac{o}{\lambda_3} z \right)$. Our average over the volume, for $m \neq 0$, using wolfram alpha to do the dirty work, is then

$$
\begin{aligned}
&\frac{1}{\lambda_1\lambda_2\lambda_3}\int_{z=0}^{\lambda_3} dz \int_{y=0}^{\lambda_2} dy \int_{x=0}^{\lambda_1} dx \sin^2\left(-\frac{2\pi m x}{\lambda_1} - \frac{2\pi n y}{\lambda_2} - \frac{2\pi o z}{\lambda_3} + \omega t\right) \\
&= \frac{1}{\lambda_1\lambda_2\lambda_3}\int_{z=0}^{\lambda_3} dz \int_{y=0}^{\lambda_2} dy \frac{-\lambda_1}{4\pi m}\left(-\frac{2\pi m}{\lambda_1}x - \frac{2\pi n y}{\lambda_2} - \frac{2\pi o z}{\lambda_3} + \omega t\right)\Bigg|_{x=0}^{\lambda_1} \\
&- \frac{1}{\lambda_1\lambda_2\lambda_3}\int_{z=0}^{\lambda_3} dz \int_{y=0}^{\lambda_2} dy \frac{-\lambda_1}{8\pi m}\sin\left(2\left(-\frac{2\pi m}{\lambda_1}x - \frac{2\pi n y}{\lambda_2} - \frac{2\pi o z}{\lambda_3} + \omega t\right)\right)\Bigg|_{x=0}^{\lambda_1}
\end{aligned}
$$

Since the sine integral vanishes, we have just $1/2$ as expected regardless of the angular frequency ω. Okay, that makes sense now. Looks like ω is only relevant for the single $\mathbf{K} = 0$ Fourier component, but that likely doesn't matter since I seem to recall that the $\mathbf{K} = 0$ Fourier component of this oscillators in a box problem was entirely constant (and perhaps zero?).

3.2. ***Third Lecture. Page 7 notes.***

The units in the transformation for the wave function don't look right. We want to transform the Pauli equation

$$
i\hbar\frac{\partial\Psi}{\partial t} = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2\Psi + e\phi\Psi,
$$

with a transformation of the form

$$
\begin{aligned}
\mathbf{A}' &= \mathbf{A} + \boldsymbol{\nabla}\chi \\
\phi' &= \phi - \frac{1}{c}\frac{\partial\chi}{\partial t} \\
\Psi' &= e^{-i\mu}\Psi,
\end{aligned}
$$

Where $\mu \propto \chi$ is presumed, and we want to find the proportionality constant required for invariance. With $\mathbf{p} = -i\hbar\boldsymbol{\nabla}$ we have

$$
\begin{aligned}
\mathbf{p}\Psi' &= -i\hbar\boldsymbol{\nabla}e^{-i\mu}\Psi \\
&= -i\hbar\left(-i(\boldsymbol{\nabla}\mu)e^{-i\mu}\Psi + e^{-i\mu}\boldsymbol{\nabla}\Psi\right) \\
&= +e^{-i\mu}\left(\mathbf{p} + \hbar\boldsymbol{\nabla}\mu\right)\Psi,
\end{aligned}
$$

so

$$
(\mathbf{p} - \frac{e}{c}\mathbf{A}')\Psi' = e^{-i\mu}\left(\mathbf{p} - \frac{e}{c}\mathbf{A} - \boldsymbol{\nabla}(\hbar\mu + \frac{e}{c}\chi)\right)\Psi.
$$

For the time partial we have

$$
\frac{\partial\Psi'}{\partial t} = e^{-i\mu}\frac{\partial\Psi}{\partial t} - i\frac{\partial\mu}{\partial t}e^{-i\mu}\Psi,
$$

and the scalar potential term transforms as

$$e\phi'\Psi' = e\left(\phi - \frac{1}{c}\frac{\partial \chi}{\partial t}\right) e^{-i\mu}\Psi$$

Putting the pieces together we have

$$i\hbar e^{-i\mu}\left(\frac{\partial}{\partial t} - i\frac{\partial \mu}{\partial t}\right)\Psi = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A} - \frac{e}{c}\boldsymbol{\nabla}\chi\right) e^{-i\mu}\left(\mathbf{p} - \frac{e}{c}\mathbf{A} - \boldsymbol{\nabla}(\hbar\mu + \frac{e}{c}\chi)\right)\Psi + e\left(\phi - \frac{1}{c}\frac{\partial \chi}{\partial t}\right) e^{-i\mu}\Psi$$

We need one more intermediate result, that of

$$\begin{aligned}
\mathbf{p}e^{-i\mu}\mathbf{D} &= -i\hbar e^{-i\mu}\left(-i(\boldsymbol{\nabla}\mu) + \boldsymbol{\nabla}\right)\mathbf{D} \\
&= e^{-i\mu}(\mathbf{p} - \hbar\boldsymbol{\nabla}\mu)\mathbf{D}.
\end{aligned}$$

So we have

$$i\hbar\frac{\partial \Psi}{\partial t} + \hbar\frac{\partial \mu}{\partial t}\Psi = \frac{1}{2m}\left(\mathbf{p} - \hbar\boldsymbol{\nabla}\mu - \frac{e}{c}\mathbf{A} - \frac{e}{c}\boldsymbol{\nabla}\chi\right)\left(\mathbf{p} - \frac{e}{c}\mathbf{A} - \boldsymbol{\nabla}(\hbar\mu + \frac{e}{c}\chi)\right)\Psi + e\left(\phi - \frac{1}{c}\frac{\partial \chi}{\partial t}\right)\Psi.$$

To get rid of the μ, and χ time partials we need

$$\hbar\frac{\partial \mu}{\partial t} = -\frac{e}{c}\frac{\partial \chi}{\partial t}$$

Or

$$\mu = -\frac{e}{c\hbar}\chi$$

This also kills off all the additional undesirable terms in the transformed $\mathbf{P}^2$ operator (with $\mathbf{P} = \mathbf{p} - e\mathbf{A}/c$), leaving the invariant transformation completely specified

$$\begin{aligned}
\mathbf{A}' &= \mathbf{A} + \boldsymbol{\nabla}\chi \\
\phi' &= \phi - \frac{1}{c}\frac{\partial \chi}{\partial t} \\
\Psi' &= \exp\left(i\frac{e}{\hbar c}\chi\right)\Psi,
\end{aligned}$$

This is a fair bit different than the final result as noted in the text, but since that starts with the wrong electrodynamic gauge transformation, this is not too unexpected.

3.3. ***Third Lecture. Page 8 notes.***

Here we have

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 - \frac{e\hbar}{2mc}(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\times\mathbf{A}) + eV$$

whereas previously it was

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2m} \left[\sigma \cdot \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) \right] \left[\sigma \cdot \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) \right] \Psi + e\phi\Psi.$$

What is this $[\sigma \cdot \mathbf{x}]$ notation? In [1] we have

$$\mathbf{a} \cdot \boldsymbol{\sigma} = a_i \sigma_i,$$

Within these square braces it appears that this product is intended to be a tensor product, like so

$$\begin{aligned}
[\sigma \cdot \mathbf{a}] [\sigma \cdot \mathbf{b}] &\stackrel{?}{=} \sum_{i,j} a_i \sigma_i b_j \sigma_j \\
&= (\mathbf{a} \cdot \mathbf{b}) I + i \sigma \cdot (\mathbf{a} \times \mathbf{b}).
\end{aligned}$$

For H this would be

$$\begin{aligned}
i\hbar \frac{\partial \Psi}{\partial t} &= \frac{1}{2m} \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) \cdot \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) - \frac{i^2 \hbar e}{2mc} \sigma \cdot (\boldsymbol{\nabla} \times \mathbf{A}) \Psi + e\phi\Psi \\
&= \frac{1}{2m} \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) \cdot \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) + \frac{\hbar e}{2mc} \sigma \cdot (\boldsymbol{\nabla} \times \mathbf{A}) \Psi + e\phi\Psi.
\end{aligned}$$

Ah. The i in $\mathbf{p} = -i\hbar \boldsymbol{\nabla}$ is what does away with the i in the Pauli matrix product. However, there does appear to be a sign error.

Instead of guessing what Feynman means when he writes Pauli's equation, it would be better to just check what Pauli says. In

Now, how does one reconsile this with Pauli's text [2] he writes

$$H = \frac{1}{2m} \sum_{k=1}^{3} \left(p_k - \frac{e}{c} A_k \right)^2 + e\phi V.$$

There is no $\boldsymbol{\nabla} \times \mathbf{A}$ operator term in Pauli's own text, just the scalar operator?

References

[1] Wikipedia. Pauli matrices — wikipedia, the free encyclopedia [online]. 2009. [Online; accessed 16-June-2009]. Available from: http://en.wikipedia.org/w/index.php?title=Pauli_matrices&oldid=296796770. 3.3

[2] W. Pauli. *Wave Mechanics*. Courier Dover Publications, 2000. 3.3