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Normal transmission and reflection through two interfaces

Motivation In class an outline of normal transmission through a slab was presented. Let's go through the details.

Normal incidence The geometry of a two interface configuration is sketched in fig. 1.1.

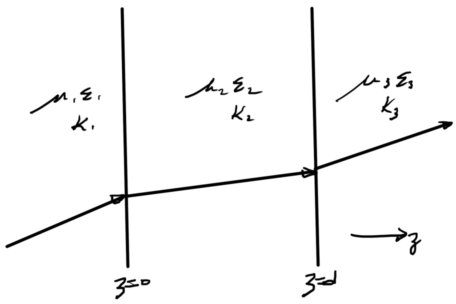

Figure 1.1: Two interface transmission.

Given a normal incident ray with magnitude A, the respective forward and backwards rays in each the mediums can be written as

I

$$\begin{aligned} &A e^{-j k_{1z} z} \\ &A r e^{j k_{1z} z} \end{aligned} \tag{1.1}$$

II

$$\begin{aligned} &C e^{-j k_{2z} z} \\ &D e^{j k_{2z} z} \end{aligned} \tag{1.2}$$

III

$$A t e^{-j k_{3z} (z - d)} \tag{1.3}$$

Matching at $z = 0$ gives

$$\begin{aligned} At_{12} + r_{21}D &= C \\ Ar &= Ar_{12} + Dt_{21}, \end{aligned} \tag{1.4}$$

whereas matching at $z = d$ gives

$$\begin{aligned} At &= Ce^{-jk_{2z}d}t_{23} \\ De^{jk_{2z}d} &= Ce^{-jk_{2z}d}r_{23} \end{aligned} \tag{1.5}$$

We have four linear equations in four unknowns r, t, C, D, but only care about solving for r, t. Let's write $\gamma = e^{jk_{2z}d}$, $C' = C/A$, $D' = D/A$, for

$$\begin{aligned} t_{12} + r_{21}D' &= C' \\ r &= r_{12} + D't_{21} \\ t\gamma &= C't_{23} \\ D'\gamma^2 &= C'r_{23} \end{aligned} \tag{1.6}$$

Solving for C', D' we get

$$\begin{aligned} D'\left(\gamma^2 - r_{21}r_{23}\right) &= t_{12}r_{23} \\ C'\left(\gamma^2 - r_{21}r_{23}\right) &= t_{12}\gamma^2, \end{aligned} \tag{1.7}$$

so

$$\begin{aligned} r &= r_{12} + \frac{t_{12}t_{21}r_{23}}{\gamma^2 - r_{21}r_{23}} \\ t &= t_{23}\frac{t_{12}\gamma}{\gamma^2 - r_{21}r_{23}}. \end{aligned} \tag{1.8}$$

With $\phi = -jk_{2z}d$, or $\gamma = e^{-j\phi}$, we have

$$\boxed{\begin{aligned} r &= r_{12} + \frac{t_{12}t_{21}r_{23}e^{2j\phi}}{1 - r_{21}r_{23}e^{2j\phi}} \\ t &= \frac{t_{12}t_{23}e^{j\phi}}{1 - r_{21}r_{23}e^{2j\phi}}. \end{aligned}} \tag{1.9}$$

A slab When the materials in region I, and III are equal, then $r_{12} = r_{32}$. For a TE mode, we have

$$r_{12} = \frac{\mu_2 k_{1z} - \mu_1 k_{2z}}{\mu_2 k_{1z} + \mu_1 k_{2z}} = -r_{21}. \tag{1.10}$$

so the reflection and transmission coefficients are

$$\begin{aligned} r^{\text{TE}} &= r_{12}\left(1 - \frac{t_{12}t_{21}e^{2j\phi}}{1 - r_{21}^2 e^{2j\phi}}\right) \\ t^{\text{TE}} &= \frac{t_{12}t_{21}e^{j\phi}}{1 - r_{21}^2 e^{2j\phi}}. \end{aligned} \tag{1.11}$$

It's possible to produce a matched condition for which $r_{12} = r_{21} = 0$, by selecting

$$\begin{aligned} 0 &= \mu_2 k_{1z} - \mu_1 k_{2z} \\ &= \mu_1 \mu_2 \left(\frac{1}{\mu_1} k_{1z} - \frac{1}{\mu_2} k_{2z} \right) \\ &= \mu_1 \mu_2 \omega \left(\frac{1}{v_1 \mu_1} \theta_1 - \frac{1}{v_2 \mu_2} \theta_2 \right), \end{aligned} \tag{1.12}$$

or

$$\frac{1}{\eta_1} \cos\theta_1 = \frac{1}{\eta_2} \cos\theta_2, \tag{1.13}$$

so the matching condition for normal incidence is just

$$\eta_1 = \eta_2. \tag{1.14}$$

Given this matched condition, the transmission coefficient for the 1,2 interface is

$$\begin{aligned} t_{12} &= \frac{2\mu_2 k_{1z}}{\mu_2 k_{1z} + \mu_1 k_{2z}} \\ &= \frac{2\mu_2 k_{1z}}{2\mu_2 k_{1z}} \\ &= 1, \end{aligned} \tag{1.15}$$

so the matching condition yields

$$\begin{aligned} t &= t_{12} t_{21} e^{j\phi} \\ &= e^{j\phi} \\ &= e^{-jk_{2z} d}. \end{aligned} \tag{1.16}$$

Normal transmission through a matched slab only introduces a phase delay.