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## A silly geometry problem: length of side of square in circular quadrant

Problem from Solving a geometry question that no one can figure out.

My solution (before numerical reduction), using basic trig and complex numbers, is illustrated in fig. 1.1.



Figure 1.1: With complex numbers.

We have

$$s = x \cos \theta$$
  

$$y = x \sin \theta$$
  

$$p = y + xe^{i\theta}$$
  

$$q = is + xe^{i\theta}$$
  

$$|q| = y + 5$$
  

$$|p - q| = 2.$$
  
(1.1)

This can be reduced to

$$\begin{vmatrix} xe^{i\theta} - 5 \end{vmatrix} = 2$$

$$x \left| i\cos\theta + e^{i\theta} \right| = x\sin\theta + 5.$$
(1.2)

My wife figured out how to do it with just Pythagoras, as illustrated in fig. 1.2.



Figure 1.2: With Pythagoras.

$$(5-s)^{2} + y^{2} = 4$$

$$(s+y)^{2} + s^{2} = (y+5)^{2}$$

$$x^{2} = s^{2} + y^{2}.$$
(1.3)

Either way, the numerical solution is 4.12. The geometry looks like fig. 1.3.

A mathematica notebook to compute the numerical part of the problem (either way) and plot the figure to scale can be found in my mathematica github repo.



**Figure 1.3:** Lengths to scale.