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Eigenvalues of 2x2 matrix: another identity seen on twitter.

Here's another interesting looking twitter math post, this time about 2x2 matrix eigenvalues:

Theorem 1.1: Eigenvalues of a 2x2 matrix.

Let m be the mean of the diagonal elements, and p be the determinant. The eigenvalues of the matrix are given by

$$m \pm \sqrt{m^2 - p}.$$

This is also not hard to verify.

Proof. Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \tag{1.1}$$

where we are looking for λ that satisfies the usual zero determinant condition

$$\begin{aligned} 0 &= \left| A - \lambda I \right| \\ &= \begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} \\ &= (a - \lambda)(d - \lambda) - bc \\ &= ad - bc - \lambda (a + d) + \lambda^2 \\ &= \det A - \lambda \operatorname{Tr} A + \lambda^2 \\ &= \left(\lambda - \frac{\operatorname{Tr} A}{2} \right)^2 + \det A - \left(\frac{\operatorname{Tr} A}{2} \right)^2, \end{aligned} \tag{1.2}$$

so

$$\lambda = \frac{\operatorname{Tr} A}{2} \pm \sqrt{ \left(\frac{\operatorname{Tr} A}{2} \right)^2 - \det A }. \tag{1.3}$$

substitution of the variables in the problem statement finishes the proof. □

Clearly the higher dimensional characteristic equation will also have both a trace and determinant dependency as well, but the cross terms will be messier (and nobody wants to solve cubic or higher equations by hand anyways.)