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What will be the value of k to satisfy this integral equation

Another problem from x/twitter ([1]):

Find k, where

$$\int_0^{2\pi} \sin^4 x dx = k \int_0^{\pi/2} \cos^4 x dx. \tag{1.1}$$

I initially misread the integration range in the second integral as 2π, not $\pi/2$, in which case the answer is just 1 by inspection. However, solving the stated problem, is not much more difficult.

Since sine and cosine are equal up to a shift by $\pi/2$

$$\sin(u + \pi/2) = \frac{e^{i(u+\pi/2)} - e^{-i(u+\pi/2)}}{2i} = \frac{e^{iu} + e^{-iu}}{2} = \cos u, \tag{1.2}$$

we can make an $x = u + \pi/2$ substitution in the sine integral.

Observe that $\cos^4 x = |\cos x|^4$, but the area under $|\cos x|$ is the same for each $\pi/2$ interval. This is shown in fig. 1.1.

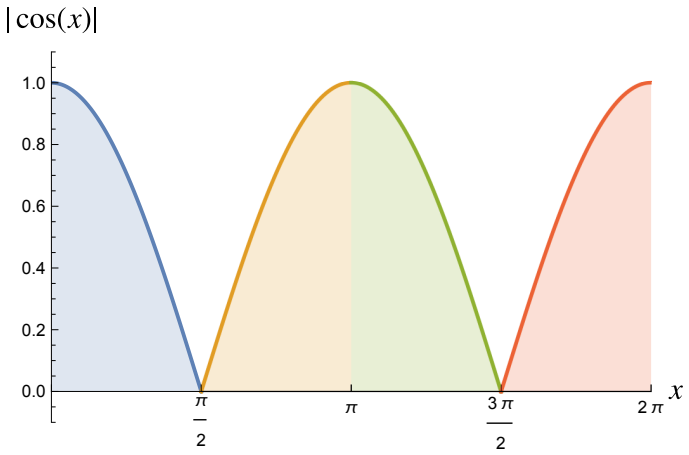

Figure 1.1: Plot of $|\cos x|$

Of course, the area under $\cos^4 x$, will also have the same periodicity, but those regions will be rounded out by the power operation, as shown in fig. 1.2.

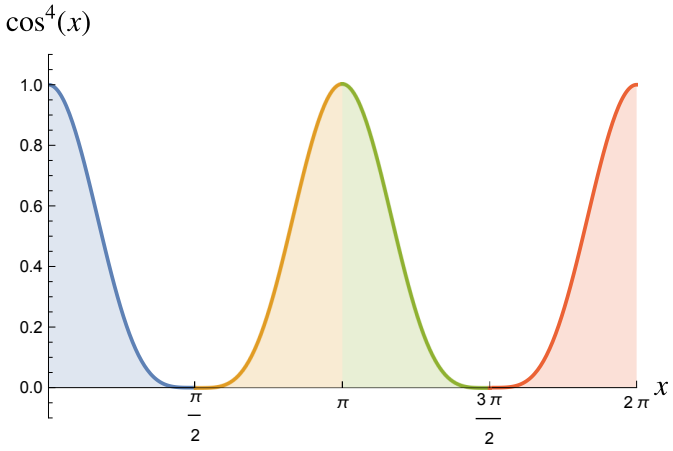

Figure 1.2: Plot of $\cos^4 x$.

Since the area under $\cos^4 x$ is the same for each $\pi/2$ wide interval, we have

$$\boxed{k = 4.} \tag{1.3}$$

Bibliography

[1] CalcInsights. *What will be the value of k to satisfy this integral equation*, 2025. URL https://x.com/CalcInsights_/status/1880932308108341443. [Online; accessed 19-Jan-2025]. 1